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Adiabatic Compression and Constant Heat Removal

220 kg of CO_2 gas at 27°C and 1 atm is compressed adiabatically to 1/5th of its volume. It is then cooled to its original temperature at constant volume. Find Q, DU and W for each step and for the entire process.

Formula:

For an ideal gas, $PV = nRT$
For an adiabatic process: PV^{γ} = constant,
$V_2/V_1 = (P_1/P_2)^{(1/\gamma)}$
$T_2/T_1 = (V_1/V_2)^{(\gamma - 1)}$
Adiabatic work of compression: $W = (P_2V_2 - P_1V_1) / (\gamma - 1) \rightarrow 1$
$Q + W = \Delta U$
$\Delta U = mC_V(T_2 - T_1) \rightarrow 2$

Calculations:

Here there are two steps taking place.

1. Adiabatic compression, and
2. Constant volume heat removal.

n = 220 / Molecular weight of CO_2 = 220/44 = 5 kmol
V_1 = 5 x 8314 x (273 + 27) / (1.01325 x 10^5) = 123.08 m^3.
V_2 = (1/5) x 123.08 = 24.616 m^3.

γ = 1.3 (for CO_2: data)
T_2 = 300 x $(5)^{0.3}$ = 486.2 K
$P_1/P_2 = (V_2/V_1)^{\gamma}$
P_2 = 1 / $(1/5)^{\gamma}$
P_2 = 8.103 atm.

Adiabatic compression work W can be calculated from equn.1 or equn.2 (since $\Delta U = Q + W$; and Q = 0 for an adiabatic process).

W = (8.103 x 1.01325 x 10^5 x 24.616 - 1.01325 x 10^5 x 123.08) / 0.3

= 25.7985 MJ

Since the temperature of the system is returned to its original state, the internal energy change of the total process is zero. Therefore, DU for the heat removal step is = -25.7985 MJ.

For the constant volume cooling step, W = 0, and ΔU = -25.7985 MJ. Therefore, Q for this step = -25.7985 MJ.

Summary:

	Q	W	ΔU
Step 1	0	25.7985 MJ	25.7985 MJ
Step 2	-25.7985 MJ	0	-25.7985 MJ
Over all	-25.7985 MJ (heat removed from the system)	25.7985 MJ (work added to the system)	0

Constant Internal Energy Process

A tank having a volume of 0.1 m^3 contains air at 14 MPa and 50°C. It is connected through a valve to a larger tank having a volume of 15 m^3, which is completely evacuated. The entire assembly is completely insulated. The valve is opened and the gas allowed to come to equilibrium in both tanks. Calculate the final pressure.

Calculations:

For this process, Q = 0 and W = 0. Therefore, $\Delta U = 0$.

$\Delta U = mC_v\Delta T = 0$.

Therefore, $\Delta T = 0$.

For an ideal gas, PV = nRT

Since T is constant, $P_1V_1 = P_2V_2$

$14 \times 0.1 = P_2 \times (0.1 + 15)$

Final pressure : $P_2 = 92.715$ KPa.

Constant Pressure Heat Addition by Resistor

A vertical cylinder with a freely floating piston contains 0.1 kg air at 1.2 bar and a small electric resistor. The resistor is wired to an external 12 Volt battery. When a current of 1.5 Amps is passed through the resistor for 90 sec, the piston sweeps a volume of 0.01 m^3. Assume (i) piston and the cylinder are insulated and (ii) air behaves as an ideal gas with C_v = 700 J/(kg.K). Find the rise in temperature of air.

Calculations:

Energy added by electric resistor = V I t = 12 x 1.5 x 90 = 1620 J

Work done = $-\int PdV = -P(V_2 - V_1) = -1.2 \times 10^5 \times \Delta V = 1.2 \times 10^5 \times 0.01 = 1200$ J

ΔU = Q + W = 1620 - 1200 = 420 J (energy added to the system and work added to the system are taken as positive.)

$\Delta U = mC_vDT$

$\Delta T = 420/(0.1 \times 700) = 6$ K.

Constant Pressure Heating

One kilogram of air is heated reversibly at constant pressure from an initial state of 300 K and 1 bar until its volume triples. Calculate W, Q, ΔU, and ΔH for the process. Assume air obeys the relation PV/T = 83.14 bar cm^3 mol^{-1} K^{-1} and that C_p = 29 J mol^{-1} K^{-1}.

Calculations:

From the relation, PV/T = 83.14 bar cm^3 mol^{-1} K^{-1}

Initial molal volume = 83.14 x 300 /1
= 24942 cm^3/mol
= 24.942 m^3/kmol

Initial volume of 1 kg of air (V_1) = 24.942 m^3 / 29 = 0.86 m^3

Final volume (V_2)= 3 x V_1 = 3 x 0.86 = 2.58 m^3 = 2.58 x 10^6 cm^3.

Final specific volume = 24942 x 3 = 74826 cm^3/mol

Final temperature:

P_2 x 74826 = T_2 x 83.14
1 x 74826 = T_2 x 83.14
T_2 = 900 K

For the constant pressure heating process:

W = -P(V_2 - V_1) = -1 x (2.58 x 10^6 - 0.86 x 10^6) bar cm^3
= -1 x 10^5 x (2.58 - 0.86) Nm
= -172000 J = -172 kJ. (work is done by the system by expansion)

Moles of air in 1 kg mass = 1000/29 = 34.483 mol

Q = mC_pDT = 34.483 x 29 x (900 - 300)
= 600000 J = 600 kJ.
ΔU = Q + W = 600 - 172
= 428 kJ (heat given to the system and work done on the system are positive quantities)
ΔH = Q = 600 kJ

Summary:

W	-172 kJ
Q	600 kJ
ΔU	428 kJ
ΔH	600 kJ

Constant Volume Heat Addition

A 28 liter rigid enclosure contains air at 140 kPa and 20°C. Heat is added to the container until the pressure reaches 345 kPa. Calculate the heat added.

Formulae:

For an ideal gas, PV = nRT

From first law of thermodynamics,

Q + W = ΔU

For an ideal gas ΔU is a function of temperature only, and work done at constant volume = 0

Therefore: $Q = \Delta U$.

$\Delta U = mC_V(T_2 - T_1)$

m = n x Molecular weight

Calculations:

n = PV / (RT) = 140 x 1000 x 0.028 / (8314 x 293)
= 1.609×10^{-3} kmol
m = 1.609×10^{-3} x 29 = 0.0467 kg.

For constant volume system: $P \alpha T$.

Therefore, $P_1/T_1 = P_2/T_2$
$140/293 = 345/T_2$
$T_2 = 722$ K.
$C_V = 0.718$ kJ/kg.°C (data, for air at 20°C)

$Q = \Delta U = 0.0467 \times 0.718 \times (722 - 293) = 14.385$ kJ

Heat added = **14.385 kJ**

Reversible Heat Engine

A heat engine receives 500 BTU of heat per cycle from a reservoir at 540°F and rejects heat to a sink at 40°F in a hypothetical amount of (a) 375 BTU per cycle (b) 250 BTU per cycle and (c) 150 BTU per cycle. Which of these respective cases represent a reversible cycle, an irreversible cycle and an impossible cycle?

Calculations:

Maximum efficiency of a heat engine = efficiency of Carnot engine

Efficiency of Carnot engine $\eta_{max} = 1 - T_2/T_1$

T1 = temperature of source = 540°F = 540 + 460 = 1000°R

T2 = temperature of sink = 40°F = 40 + 460 = 500°R

$\eta_{max} = 1 - 500/1000 = 0.5$

(a) Heat rejected to the sink = 375 BTU

$\eta_a = 1 - Q_2/Q_1 = 1 - 375/500 = 0.25$

(b) Heat rejected to the sink = 250 BTU

$\eta_b = 1 - 250/500 = 0.5$

(c) Heat rejected to the sink = 150 BTU

$\eta_c = 1 - 150/500 = 0.7$

From the above calculations we can see that the case 'c' is an impossible one (since the efficiency of a heat engine can not be more than that of Carnot cycle efficiency).

Summary:

Case: a - irreversible cycle

Case: b - reversible cycle

Case: c - impossible cycle

Enthalpy Change of Water

Liquid water at 100°C and 1 bar has an internal energy (on an arbitrary scale) of 419 kJ/kg and a specific volume of 1.044 cm^3/g.

(a) What is its enthalpy?

(b) The water is brought to the vapor state at 200°C and 800 kPa, where its enthalpy is 2838.6 kJ/kg and its specific volume is 260.79 cm^3/g. Calculate DU and DH for the process.

Calculations:

a. Enthalpy = U + PV

= 419 + 100 kPa x 1.044 x 10^{-3} m^3/kg

= 419 + 0.1044

= 419.1044 kJ/kg

b. Internal energy of water vapor at 200°C and 800 kPa = H - PV

= 2838.6 - PV = 2838.6 - 800 x 260.79 x 10^{-3}

= 2838.6 - 208.632

= 2629.968 kJ/kg

ΔU = 2629.968 - 419 = 2211.968 kJ/kg

ΔH = 2838.6 - 419.1044 = 2419.4956 kJ/kg

Entropy Change in Heat Exchange of Liquid

A heat exchanger uses 5000 kg/hr of water to cool hydrocarbon oil from 140°C to 65°C. The oil, flowing at the rate of 2500 kg/hr has an average specific heat of 0.6 kcal/kg.°C. The water enters at 20°C. Determine

i. The entropy change of the oil
ii. Entropy change of water
iii. The total entropy change as a result of this heat exchange process.

Calculations:

By energy balance: $m_{oil}C_{Poil}\Delta T_{oil} = m_{water}C_{Pwater}\Delta T_{water}$

$$\Delta T_{water} = 2500 \times 0.6 \times (140 - 65) / (5000 \times 1) = 22.5^{o}C$$

Therefore, exit temperature of water = 20 + 22.5 = 42.5°C

Entropy change is given by

$$\Delta S = mC_P \ln (T_2/T_1)$$

where m is mass flow rate of stream, T_1 and T_2 are respectively initial and final temperatures.

(i) Entropy change of oil:

T_1 = 140 + 273 = 413 K
T_2 = 65 + 273 = 338 K
Mass flow rate of oil = 2500 kg/hr = 0.6944 kg/sec
ΔS_{oil} = 0.6944 x 0.6 x ln (338/413)
= -0.0835 kcal/sec.°C

(ii) Entropy change of water:

T_1 = 20 + 273 = 293 K
T_2 = 42.5 + 273 = 315.5 K
Mass flow rate of water = 5000 kg/hr = 1.389 kg/sec
ΔS_{water} = 1.389 x 1 x ln (315.5/293)
= 0.1028 kcal/sec.°C

(iii) Total entropy change:

Total entropy change is the sum of the entropy changes of two streams
= -0.0835 + 0.1028 = 0.0193 kcal/sec.°C

Entropy Change of Air

Calculate the change in entropy when 10 kg of air is heated at constant volume from a pressure of 101325 N/m^2 and a temperature of 20°C to a pressure of 405300 N/m^2. C_V = 20.934 kJ/kmol.°C.

Calculations:

Entropy change of an ideal gas,

$$\Delta S = C_p \ln (T_2/T_1) - R \ln (P_2/P_1)$$

T_1 = 20°C = 293 K

P_1 = 101325 N/m^2

P_2 = 405300 N/m^2

For the constant volume heating process, $V_2 = V_1$

Therefore, $P_2V_1 = RT_2$

$V_1 = RT_1/P_1 = 293R/101325$

Therefore, T_2 =405300 x (293R/101325) /R = 293 x (405300/101325)

= 1172 K

$C_p = R + C_v$ = 8.314 + 20.934 = 29.248 kJ/kmol.°C

ΔS = 29.248 x ln (1172/293) - 8.314 x ln (405300/101325)

= 40.546 - 11.526 = 29.02 kJ/kmol.°C

kmol of air equivalent to 10 kg of air = 10/29 = 0.345 kmol

Change in entropy of air = 0.345 x 29.02 = 10.007 kJ/°C

Entropy Change of Heat Exchanger

An ideal gas C_P = 7R/2 is heated in a steady flow heat exchanger from 70°C to 190°C, by another stream of the same ideal gas entering at 320°C. The flow rates of the two streams are the same.

(i) Calculate ΔS of the two gas streams for countercurrent flow
(ii) What is ΔS_{total}?
(iii) If heating stream enters at 200°C, what is ΔS_{total}?

Calculations:

Basis: 1 mole of gas

Entropy change for the constant pressure heating/cooling process:

$$\Delta S = nC_P \ln (T_2/T_1)$$

Where T_1 and T_2 are respectively initial and final temperatures.

(i) *Entropy change of the cold gas stream:*

$T_1 = 273 + 70 = 343$ K
$T_2 = 273 + 190 = 463$ K
n = 1

Therefore, $\Delta S = (7R/2) \times \ln (463/343) = 1.05$ R

Entropy change of the Hot gas stream:

$T_1 = 273 + 320 = 593$ K

Since the flow rates of the two streams are same,

$$\Delta T_{hotstream} = \Delta T_{coldstream} = 190 - 70 = 120 \text{ K}$$

$$T_2 = T_1 - 120 = 593 - 120 = 473 \text{ K}$$

n = 1

Therefore, $\Delta S = (7R/2) \times \ln (473/593) = -0.79$ R

(ii) $\Delta S_{total} = 1.05 R + (-0.79R) = 0.26$ R
(iii) *Heating stream inlet temperature = 200°C*

Therefore, outlet temperature of heating stream = 200 - 120 = 80°C

ΔS for this stream = (7R/2) x ln (353/473) = -1.024 R
$\Delta S_{total} = 1.05 R + (-1.024 R) = 0.026$ R

Entropy Change of Mixing of Two Gases

What is the change in entropy when 0.7 m^3 of CO_2 and 0.3 m^3 of N_2 each at 1 bar and 25°C blend to form a homogenous mixture at the same conditions? Assume ideal gases.

Calculations:

Since there is no change in temperature after mixing, internal energy change of individual gases is zero.

For a reversible process, dU = TdS - PdV

And for constant internal energy process dU = 0. Therefore TdS = PdV

$$dS = PdV/T \qquad (1)$$

For an ideal gas: PV = nRT

Therefore, P/T = nR/V

Substituting for P/T in Equation 1,

$$dS = nR\, dV/V$$

Integrating the above equation,

$$\Delta S = nR \ln(V_2/V_1) \qquad (2)$$

Since mixing is an irreversible process, we can calculate the entropy change of individual gases by assuming the equivalent reversible process given below:

(a) Reversible expansion of gas from its initial volume to the final volume of 1 m^3. Entropy change for this process is given by Equation 2

(b) Mixing two gases at identical conditions. (each at a volume of 1 m^3). Since there is no change in thermodynamic conditions in this process, entropy change is zero for this step. (Entropy is a state function)

We shall calculate the entropy change of individual gases and total entropy change of the system is the sum of the entropy changes of two gases.

For CO_2:

Moles of $CO_2 = 10^5 \times 0.7 / (R \times 298) = 234.9 / R$

$\Delta S = 234.9 \ln (1/0.7) = 83.78$ J/°C

For N_2:

Moles of $N_2 = 10^5 \times 0.3 / (R \times 298) = 100.7 / R$

$\Delta S = 100.7 \ln (1/0.3) = 121.24$ J/°C

Entropy change of the system = 83.78 + 121.24 = 205.02 J/°C

Entropy Change – Adiabatic Process

A lump of copper having a mass of 10 gm at a temperature of 500°C is just dropped into a well insulated bucket containing 100 gm of water at a temperature of 50°C. If the heat capacities of copper and water are 0.095 and 1.0 respectively, expressed as cal/gm.°C, calculate the total change in entropy resulting from the process.

Calculations:

Final temperature of copper and water will be the same, and is obtained by enthalpy balance.

Heat lost by copper = heat gained by water

$[mC_P(T_1 - T_2)]_{Copper} = [mC_P(T_2 - T_1)]_{Water}$

$10 \times 0.095 \times (500 - T_2) = 100 \times 1 \times (T_2 - 50)$
$0.95 \times (500 - T_2) = 100 \times (T_2 - 50)$
$(500 - T_2) = 105.26 \times (T_2 - 50)$
$500 + 5263.2 = 106.26\ T_2$
$T_2 = 54.2^oC$

Entropy change is given by

$\Delta S = m\ C_P \ln (T_2/T_1)$

Where m is mass; C_P is specific heat; T_1 and T_2 are initial and final temperatures respectively.

Entropy change of copper:

$T_1 = 273 + 500 = 773$ K
$T_2 = 273 + 54.2 = 327.2$ K

$\Delta S_{Copper} = 10 \times 0.095 \times \ln (327.2/773) = -0.817$ cal/^{o}C

Entropy change of water:

$T_1 = 273 + 50 = 323$ K
$T_2 = 273 + 54.2 = 327.2$ K

$\Delta S_{Water} = 100 \times 1 \times \ln (327.2/323) = 1.292$ cal/^{o}C

Total entropy change = $\Delta S_{Copper} + \Delta S_{Water} = -0.817 + 1.292 = 0.475$ cal/^{o}C

First Law – Cycle Work

A thermodynamic system undergoes a cycle composed of a series of three processes for which Q1 = +10 kJ, Q2 = +30 kJ, Q3 = -5 kJ. For the first process, ΔE = +20 kJ, and for the third process, ΔE = -20 kJ. What is the work in the second process, and the net work output of the cycle?

Calculations:

$$Q + W = \Delta E$$

Work done in the first process = 20 - 10 = 10 kJ (i.e., work is done on the system)

Work done in the third process = -20 - (-5) = -15 kJ

For a cyclic process, the overall internal energy change is zero. (i.e., $\Delta E = 0$)

Therefore, ΔE in the second process = (0 - (20 - 20)) = 0 kJ

Therefore, work done in the second process = 0 - 30 kJ = **-30 kJ**.

Total work done during the cycle = 10 + (-15) + (-30) = **-35 kJ** (i.e., 35 kJ of work is done by the system).

Total work done can also be calculated as follows:

Total Q = Q1 + Q2 + Q3 = 10 + 30 - 5 = 35 kJ

Therefore, net work done during the cycle = 0 - 35 kJ = -35 kJ (the negative sign indicates that work is done by the system).

Heat Transferred through a Coil

Water flows through a horizontal coil heated from outside. During its passage, it changes state from liquid at 200 kPa and 80°C to vapor at 100 kPa and 125°C. The entering and exit velocities are 3 m/s and 200 m/s respectively. Determine the heat transferred through the coil per unit mass of water. H_{inlet} = 334.9 kJ/kg; H_{outlet} = 2726 kJ/kg.

Calculations:

For the steady flow process, the first law is written as

$$\Delta H + \Delta u^2/2 + g\Delta z = Q + W_s$$

since there is no shaft work, $W_s = 0$

and flow is horizontal, $\Delta z = 0$

Therefore,

$$\Delta H + \Delta u^2/2 = Q$$

substituting for the quantities,

$$(2726 - 334.9) \times 1000 + (200^2 - 3^2)/2 = Q \text{ (in terms of J/kg)}$$

$$Q = 2411.1 \text{ kJ/kg}$$

Heat transferred through the coil per unit mass of water = 2411.1 kJ

Heating at Constant Volume followed by Cooling at Constant Pressure

Air is compressed from 2 atm absolute and 28°C to 6 atm absolute and 28°C by heating at constant volume followed by cooling at constant pressure. Calculate the heat and work requirements and DE and DH of the air.

Calculations:

$$PV = nRT$$

We have to rise the pressure to 6 atm by heating at constant volume.

At constant volume: $P \alpha T$.

Therefore: $T_2/T_1 = P_2/P_1$

$T_2 = (6/2) \times (273 + 28) = 903$ K

For a constant volume process, $W = 0$ and $\Delta E = Q$

$C_V = 0.718$ kJ/kg.°C (for air; data)
$Q = mC_V(T_2 - T_1) = 0.718 \times (903 - 301) = 432.24$ kJ/kg.
$\Delta E = 432.24$ kJ/kg

For cooling at constant pressure, heat removed = $mC_P(T_2 - T_1)$

$= 1.005 \times (903 - 301) = 605.01$ kJ/kg ($C_p - C_v = R$ for ideal gas)

Internal energy change for the total process consisting the above two steps is zero (since internal energy is a function of temperature alone).

Work done on the gas during constant pressure cooling = 605.01 - 432.24

= 172.77 kJ/kg.

Summary:

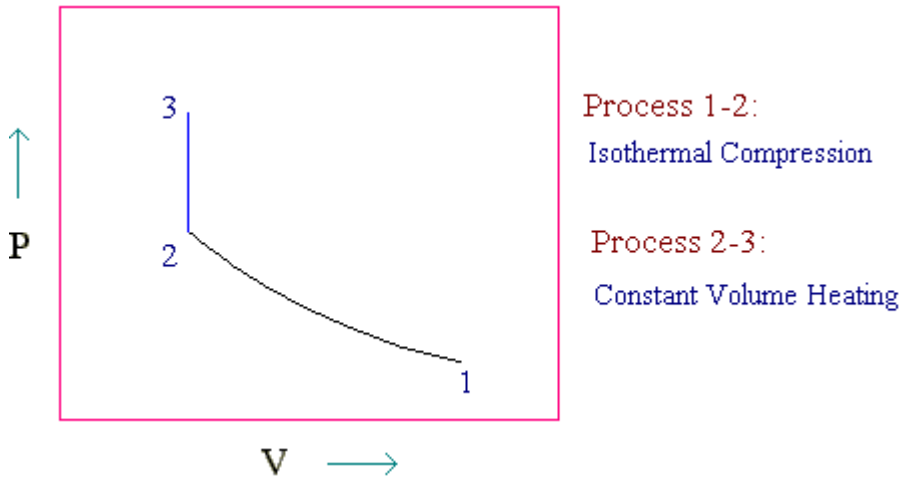
	Heat required	Work required	ΔE	ΔH
Constant Volume heat addition	432.24 kJ/kg		432.24 kJ/kg	605.01 kJ/kg
Constant Pressure heat removal	-605.01 kJ/kg	172.77 kJ/kg	-432.24 kJ/kg	-605.01 kJ/kg
Over All	-172.77 kJ/kg	172.77 kJ/kg	0	0

Isothermal Compression and Isochoric Heating

One mole of an ideal gas, initially at 20°C and 1 bar, undergoes the following mechanically reversible changes. It is compressed isothermally to a point such that when it is heated at constant volume to 100°C its final pressure is 10 bar. Calculate Q, W, ΔU, and ΔH for the process. Take $C_p = 7R/2$ and $C_v = 5R/2$.

Calculations:

The given process is represented in PV diagram as follows:



We can find the pressure and volume of points 1, 2 and 3 by equation of state (ideal gas equation)

For one mole of ideal gas,

$$P_1V_1 = RT_1$$

$$P_2V_2 = RT_2$$

$$P_3V_3 = RT_3$$

For the given process, $T_2 = T_1$ and $V_3 = V_2$.

$$V_1 = R \times (273 + 20)/1 = 293R$$

$$V_3 = R \times (273 + 100) / 10 = 37.3R$$

For the point 2, $P_2 = R \times (273 + 20) / (37.3R) = 293/37.3 = 7.855$ bar

PVT conditions of the points 1,2 and 3's are:

Point	P, bar	V	T, Kelvin
1	1	293R	293
2	7.855	37.3R	293
3	10	37.3R	373

For the isothermal process:

$\Delta U = 0$

$\Delta H = 0$

$W = \int PdV = \int RT\, dV/V = RT \ln (V_2/V_1) = R \times 293 \times \ln (37.3/293) = -604R$

$Q = -W = 604R$ (since $Q + W = \Delta U$)

Work is done on the system and heat is given out by the system

For the constant volume heating (isochoric process):

$W = 0$
$\Delta U = C_v(T_3 - T_2) = (5R/2) \times (373 - 293) = 200R$
$\Delta H = C_p(T_3 - T_2) = (7R/2) \times (373 - 293) = 280R$
$Q = \Delta U - W = 200R$

Summary:

Process	Q (heat given to the system)	W (work done on the system)	ΔU	ΔH
Isothermal compression	-604R = -604 x 8.314 = -5021.66 J	604R = 5021.66 J	0 J	0 J
Constant volume heating	200R = 1662.8 J	0 J	1662.8 J	280R = 2327.92 J

PVT-z Data

Nitrogen gas at a pressure of 100 Bar and -70°C is contained in a tank of 0.25 m^3. Heat is added until the temperature is 37°C. Determine approximately, through the use of Z factor,

(a) Specific volume of gas, in m^3/kmol
(b) The final pressure, in Bar.

Solution:

Data for N_2:

Pc = 33.9 Bar
Tc = 126.2 K

The value of universal gas constant, consistent with the system of units in this problem is,

$R = 1.01325 \times 22.4/273 = 0.0831$ Bar.m^3/(kmol.K)

(a) Molar Volume of Gas:

Initial conditions:

P_1 = 100 Bar
T_1 = (273 - 70) K
V_1 = 0.3 m^3

$P_{r1} = P_1/Pc = 100/33.9 = 2.9499$
$T_{r1} = T_1/Tc = 203/126.3 = 1.6082$

Interpolating the value of Z for the T_{r1} and p_{r1} values, Z = 0.846

Molar volume of gas (v_1) is calculated as follows: **$v_1 = Z \times R \times T_1/P_1$** = 0.8460 x 0.0831 x 203 /100 = **0.1428 m^3/kmol**

(a) Molar Volume of Gas:

Final conditions:

T2 = (273 + 37) K
$v_2 = v_1$

$P_2v_2/(RT_2) = Z_2$.

In the above equation, both pressure and Z are unknowns.

To calculate Z at the final conditions, the following iterative procedure is used:

(i) Assume a value of pressure ($P_{assumed}$). For the first iteration the assumption shall be the ideal gas value (P_{ideal}) obtained from the relation $Pv = RT$.
(ii) Using the assumed pressure of step (i), calculate Tr, Pr and the corresponding value of Z from tables/charts.
(iii) Evaluate P from $P_{calculated} = Z \times P_{ideal}$.
(iv) If the calculated pressure of step (iii), is equal to that of that assumed in step (i), the problem is essentially solved, (i.e., the assumed pressure is the answer to the problem), else repeat steps (i) to (iii) till $P_{assumed}$ is nearly equal to $P_{calculated}$.

The above procedure is applied to this problem and the calculations are as below:

Iteration 1:

$P_{assumed} = P_{ideal} = R \times T_2/v_2 = 180.5$ Bar

Pr = 180.5/33.9 = 5.3246; Tr = 310/126.2 = 2.4564.

For these Pr and Tr values, Z = 1.0432.

Therefore, $P_{calculated} = 1.0432 \times 180.5 = 188.3$ Bar.

Since $P_{calculated} <> P_{assumed}$, let us assume another pressure value say, 185 Bar.

Iteration 2:

$P_{assumed} = 185$ Bar
Z = 1.0464
$P_{calculated} = 188.88$ Bar

Iteration 3:

$P_{assumed} = 190$Bar
Z = 1.0499
$P_{calculated} = 189.51$ Bar

Iteration 4:

$P_{assumed} = 189.5$Bar
Z = 1.0495
$P_{calculated} = 189.4$ Bar

In the fourth iteration, 'assumed' and 'calculated' pressures are almost equal. So the pressure at the final conditions when T = 37°c = **189.5 Bar**

Reversible Heat Engine

A heat engine receives 500 BTU of heat per cycle from a reservoir at 540°F and rejects heat to a sink at 40°F in a hypothetical amount of (a) 375 BTU per cycle (b) 250 BTU per cycle and (c) 150 BTU per cycle. Which of these respective cases represent a reversible cycle, an irreversible cycle and an impossible cycle?

Calculations:

Maximum efficiency of a heat engine = efficiency of Carnot engine

Efficiency of Carnot engine $\eta_{max} = 1 - T_2/T_1$

T1 = temperature of source = 540°F = 540 + 460 = 1000°R

T2 = temperature of sink = 40°F = 40 + 460 = 500°R

$\eta_{max} = 1 - 500/1000 = 0.5$

(a) Heat rejected to the sink = 375 BTU

$\eta_a = 1 - Q_2/Q_1 = 1 - 375/500 = 0.25$

(b) Heat rejected to the sink = 250 BTU

$\eta_b = 1 - 250/500 = 0.5$

(c) Heat rejected to the sink = 150 BTU

$\eta_c = 1 - 150/500 = 0.7$

From the above calculations we can see that the case 'c' is an impossible one (since the efficiency of a heat engine can not be more than that of Carnot cycle efficiency).

Summary:

Case: a - irreversible cycle

Case: b - reversible cycle

Case: c - impossible cycle

Thermal Efficiency of Power Plant

A power plant operates with a heat source reservoir at 350°C and a heat sink reservoir at 30°C. It has a thermal efficiency of 55% of the Carnot engine thermal efficiency for the same temperature levels. What is the thermal efficiency of the plant?

Calculations:

Given:

Temperature of source, $T_1 = 350°C = 350 + 273 = 623$ K

Temperature of sink, $T_2 = 30°C = 30 + 273 = 303$ K

$$\eta_{Carnot} = 100 \times (1 - T_2/T_1) = 100 \times (1 - 303/623) = 48.64\%$$

Thermal efficiency of the plant = 55% of the Carnot engine efficiency (given)

$$= 0.55 \times 48.64 = 26.75\%$$

Temperature Change due to Enthalpy Change

Liquid water at 25°C flows in a straight horizontal pipe, in which there is no exchange of either heat or work with the surroundings. Its velocity is 12 m/s in a pipe with an i.d. of 2.5 cm until it flows into a section where the pipe diameter increases to 7.5 cm. What is the temperature change?

Calculations:

For the steady flow process, the first law is written as

$$\Delta H + \Delta u^2/2 + g\Delta z = Q + W_s$$

Since there is no shaft work: $W_s = 0$

And flow is horizontal: $\Delta z = 0$

Therefore,

$$\Delta H + \Delta u^2/2 = Q$$

And since there is no heat transfer: $Q = 0$

Therefore: $\Delta H + \Delta u^2/2 = 0$

Applying continuity equations,

$$u_1 A_1 \rho_1 = u_2 A_2 \rho_2$$

For water: $\rho_1 = \rho_2$

Therefore,

$u_2/u_1 = A_1/A_2 = d_1^2/d_2^2 = 2.5^2/7.5^2 = 0.1111$
$u_2 = 12 \times 0.1111 = 1.333$ m/s
$(u_2^2 - u_1^2) / 2 = (1.333^2 - 12^2) / 2 = -71.11$

Therefore, $\Delta H = 71.11$ J/kg

Enthalpy change per kg mass = 71.11 J

We know, $\Delta H = mC_p\Delta T$ and, m = 1000 g; $C_p = 4.184$ J/g.°C

Therefore,

$$\Delta T = 71.11/(1000 \times 4.184) = 0.017^{o}C$$

Temperature change = 0.017°C

Txy Diagram

From the following vapor pressure data, construct the temperature - composition diagram at 1 atm, for the system benzene-toluene, assuming ideal solution behavior.

Temperature °C	Vapor pressure, mm Hg	
	Benzene	Toluene
80	760	300
92	1078	432
100	1344	559
110.4	1748	760

Calculations:

Mole fraction in liquid phase (x) is given by,

$$x = (p_t - p_B)/(p_A - p_B)$$

The corresponding equilibrium mole fraction in vapor phase (y) is,

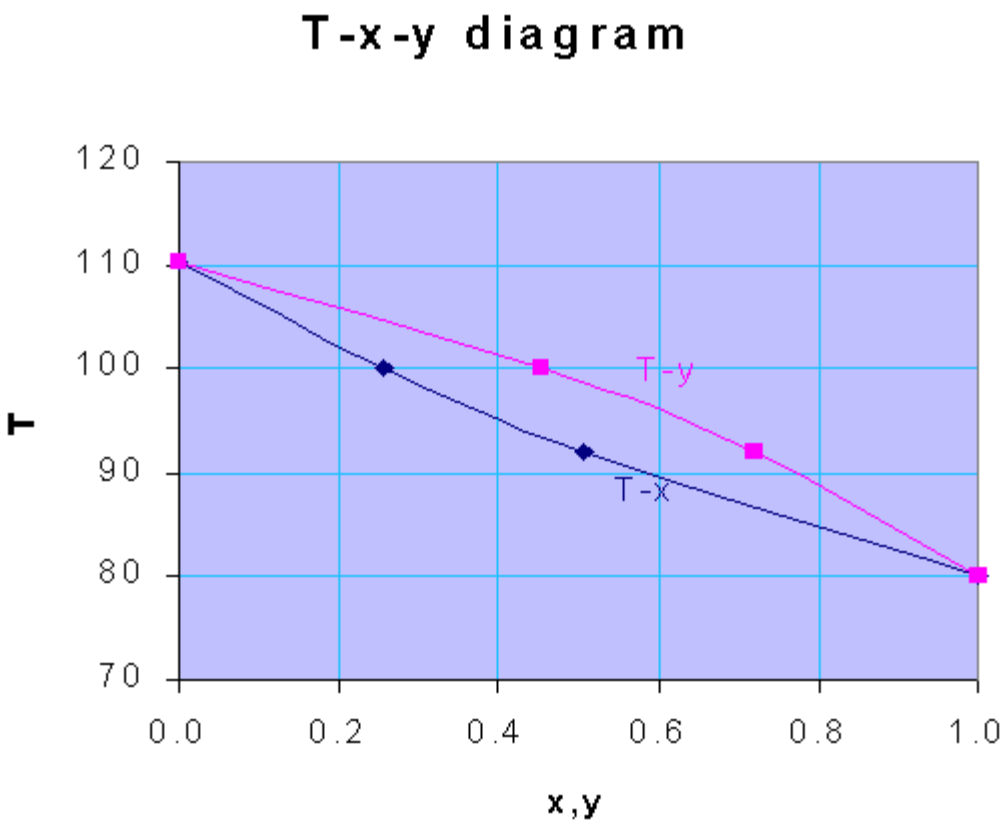
$$y = p_A x/p_t$$

Using these equations, for the various temperatures given, the following data for x and y are obtained, for the total pressure of 1 atm

T	x	y
80	1.000	1.000
92	0.508	0.720
100	0.256	0.453

110.4	0.000	0.000

From the above-calculated data, the following T-x-y diagram is drawn.



Triple Point of Ammonia

In the vicinity of the triple point, the vapor pressures of liquid and solid ammonia are respectively given by

$$\ln p = 15.16 - 3063/T \text{ and } \ln p = 18.70 - 3754/T$$

where p is in atmospheres and T is in Kelvin. What is the temperature at the triple point? Also calculate the triple point pressure.

Calculations:

At the triple point, all the phases (solid, liquid and vapor) exist in equilibrium.

Therefore, vapor pressure of liquid ammonia = vapor pressure of solid ammonia

$$15.16 - 3063/T = 18.7 - 3754/T$$

$$3754/T - 3063/T = 18.7 - 15.16$$

$3754 - 3063 = T(18.7 - 15.16)$

$691 = 3.54T$

$T = 691/3.54 = 195.2$ K

Triple point temperature = 195.2 K

Triple point pressure can be obtained from any of the vapor pressure to temperature relation.

By the vapor pressure to temperature relation of liquid ammonia,

$\ln p = 15.16 - 3063/195.2$

$p = 0.588$ atm

Similarly, by the vapor pressure to temperature relation of solid ammonia,

$\ln p = 18.7 - 3754/195.2$

$p = 0.588$ atm

Triple point pressure = 0.588 atm

Two Steps Process

An existing process consists of two steps:

(a) One mole of air $T_1 = 900$ K and $P_1 = 3$ bar is cooled at constant volume to $T_2 = 300$ K.

(b) The air is then heated at constant pressure until its temperature reaches 900 K.

It is proposed to replace this two-step process by a single isothermal expansion of the air from 900 K and 3 bar to some final pressure P. What is the value of P that makes the work of the proposed process equal to that of the existing process? Assume mechanical reversibility and treat air as an ideal gas with $C_p = 7R/2$ and $C_v = 5R/2$

Calculations:

Work in the original process consisting of two steps:

(a) For the constant volume process:

$W = 0$

Pressure at the end of the constant volume process (P_2):

$P_1V_1 = RT_1$

$V1 = R \times 900 / 3 = 300R$
$P_2V_2 = RT_2$
Since $V_1 = V_2$,
$P_2 = R \times 300 / (300R) = 1$ bar

(b) For the constant pressure process:

$P_2 = 1$ bar; $T_2 = 300$ K; $V_2 = 300R$
$P_3 = 1$ bar; $T_3 = 900$ K
$P_3V_3 = RT_3$
$V_3 = R \times 900 / 1 = 900R$
$W = \int PdV = P_2(V_3 - V_2) = 1 \times (900R - 300R) = 600R$

Total work done in the two step process = 600R

For the single step isothermal process:

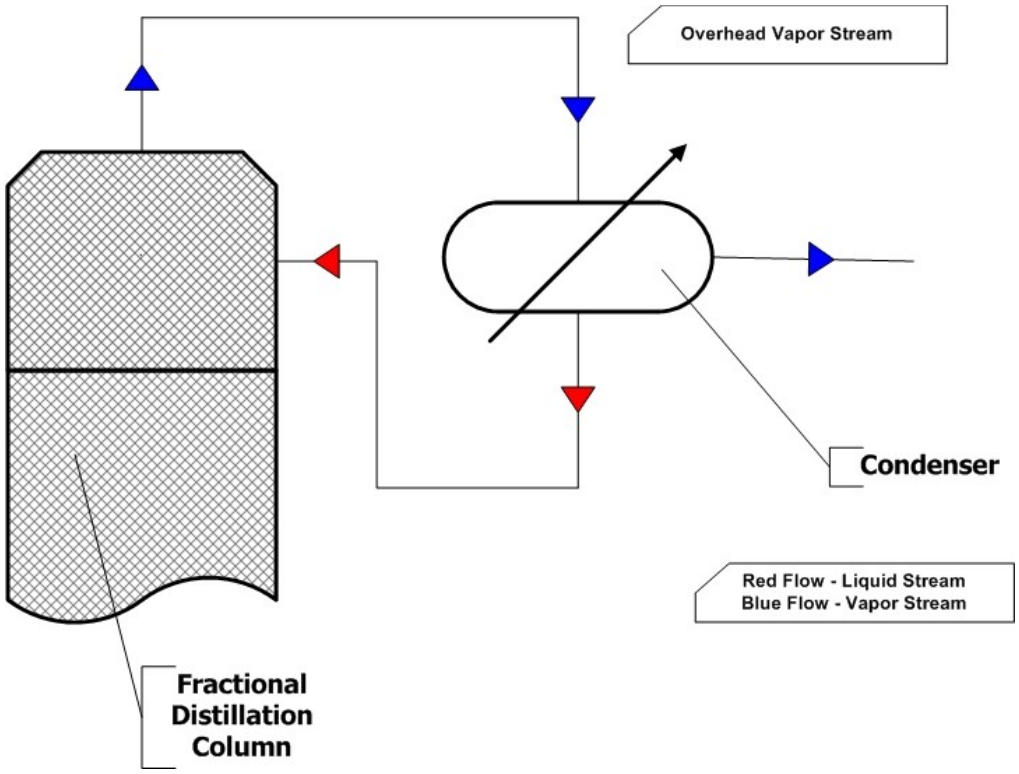
$W = 600R = RT \ln (P_1/P)$
$600R = R \times 900 \times \ln (3/P)$
$\ln(3/P_2) = 600/900 = 2/3$
$e^{(2/3)} = 3/P$
$1.9477 = 3/P$
$P = 3/1.9477 = 1.54$ bar

Problem 1

Note that for doing the following problem, DePriester Charts are required. A chart can be obtained by going to the following link and downloading and printing the DePreister Chart.

(Chart graphics courtesy University of Melbourne.)

Problem Definition



A distillation column is producing overhead vapor emissions. Analysis suggests that the emissions have the following composition:

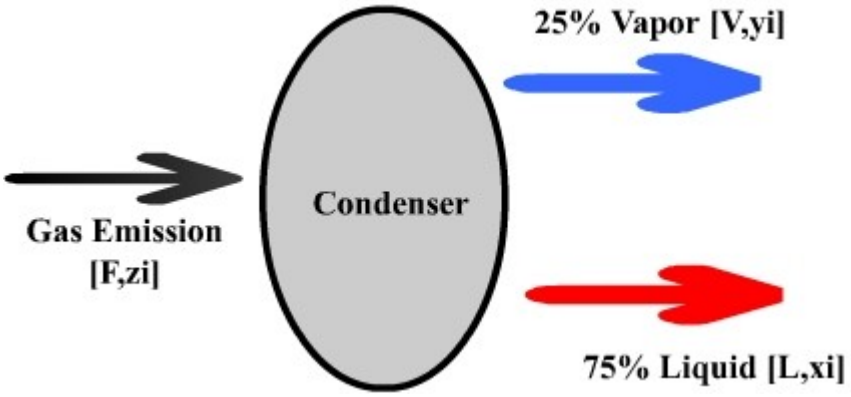
Ethane: 15%
Propane: 20%
IsoButane: 60%
nButane: 5%

Currently, the company in charge is running the condenser at 80°F and 75% of the vapor has to be condensed. You have to decide the operating pressure condition.

Hint: Iteration is required.

Solution 1:

Look at the following diagram:



Here F,V and L represent flow rates and zi, yi and xi represent component fractions.

Using mass and component balances, we can simply write:

$$F = L + V$$

$$F \cdot z_i = L \cdot x_i + V \cdot y_i$$

Also:

$$y_i = K \cdot x_i$$

Eliminate y_i and combine the top equations to obtain:

$$x_i = \frac{z_i}{\left(\frac{L}{F}\right) + \left(1 - \left(\frac{L}{F}\right)\right) \cdot K_i}$$

Now the problem is comparatively trivial, we know the incoming component fractions (z), and that (L/F) = .75 and thus we need to find the corresponding K values to obtain the component fractions in the liquid. Since the sum of all component fractions sums to unity, we ...

1 - Choose a Pressure, p.

2 - Evaluate the Ki 's for p and 80°F using the DePriester Charts.

3 - Evaluate xi's.

4 - Sum all the xi's and check if sum is unity. If not...choose a new p and redo.

A sample run is tabulated below.

		P = 100 psia		**P = 110 psia**	
Component	**Zi**	**Ki**	**Xi**	**Ki**	**Xi**
Ethane	.15	4.53	.08	4.15	**.084**

Propane	.2	1.46	.18	1.35	**.184**
isoButane	.6	.59	.669	.54	**.677**
nButane	.05	.42	.058	.38	**.059**
Sum of Xi's			.987		**1.004**

This is the best we can do (with the DePriester Charts). We can say that the pressure required is between 100 and 110 psia - closer to 110 psia.

Problem 2

You have noticed that during one of the numerous processes in a proposed chemical processing plant, ethane comes in contract with n-Hepanol at 20 atms and 25°C. You want to know whether ethanol is soluble enough at these conditions to cause some concern or whether it is completely insoluble. All you know is:

$$X_e = .0159 \quad @P = 1atm$$
$$\ln \gamma_e = B(1 - x_e)^2$$
$$K = 27.0 \quad @P = 1atm \quad \& \quad T = 25°C$$
$$K = 1.62 \quad @P = 20atm \quad \& \quad T = 25°C$$

n-Heptanol is considered non-volatile.

Solution 2

Since we are working with high enough pressures we transform...

$$P \cdot y_e = P_e^s \cdot x_e \cdot \gamma_e$$

into..

$$f_e^G \cdot y_e = f_e^L \cdot x_e \cdot \gamma_e$$

and we re-write the last equation as follows...

$$y_e = \frac{f_e^L}{f_e^G} \cdot x_e \cdot \gamma_e = K_e \cdot x_e \cdot \gamma_e$$

Now the pivotal part of the problem is that n-Heptanol is non-volatile. This means that all of the vapor is ethanol. Thus...

at 1 atm.

$$1 = (27.0)(.0159)(\gamma_e)$$

$$\gamma_e = 2.33$$

now we can solve for B

$$B = \frac{\ln(2.33)}{(1-x_e)^2} = .859$$

Thus, now we can estimate the liquid fraction x_e at 20 atms.

$$1 = 1.62 \cdot x_e \cdot e^{B(1-x_e)^2}$$

Taking natural log of both side...

$$0 = \ln(1.62) + \ln(x_e) + .859 \cdot (1-x_e)^2$$

$$x_e = 0.50$$

Thus, the ethane is pretty soluble under these conditions and thus you need to take any additional measures to ensure controllability if necessary.

Problem 3

Being the consultant of an upcoming natural gas company, you receive the following fax message from the company...

To:
IT Consulting

URGENT:
Have: **std heat of combustion of** $CH_4 = -802{,}625\ kJ/kmol$
heat of formation of $H_2O = -483{,}636\ kJ/kmol$
heat of formation of $CO_2 = -393{,}509\ kJ/kmol$

NEED h_f° **(std. heat of formation) of** CH_4.

Joseph & Motard Inc.

Can you do it?

Solution 3

What we want to find is the heat of reaction of the formation of methane...ie. the heat of reaction of...

$$C_{(s)} + 2H_{2(g)} \rightarrow CH_{4(g)}$$

Now let's begin by writing down the three relevant reactions...

Combustion of methane

Formation of carbon dioxide

Formation of water

$$CO_{2(g)} + 2H_2O \rightarrow CH_{4(g)} + 2O_{2(g)} \qquad \Delta H_{298}^\circ = 802{,}625$$
$$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)} \qquad \Delta H_{298}^\circ = -393{,}509$$
$$2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O_{(g)} \qquad \Delta H_{298}^\circ = -483{,}636$$

Now if you add the three reactions you will end up with ...

$$C_{(s)} + 2H_{2(g)} \rightarrow CH_{4(g)}$$

the very reaction you wanted! So...to get the heat of formation of methane you simply add up the 3 heats od rxns and get -74,520 kJ/kmol as the heat of formation of methane!